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Novel Topology Optimization Based on On-Off Method and Level Set Approach

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Outline of the presentation

I. Background and purpose

II. Present method

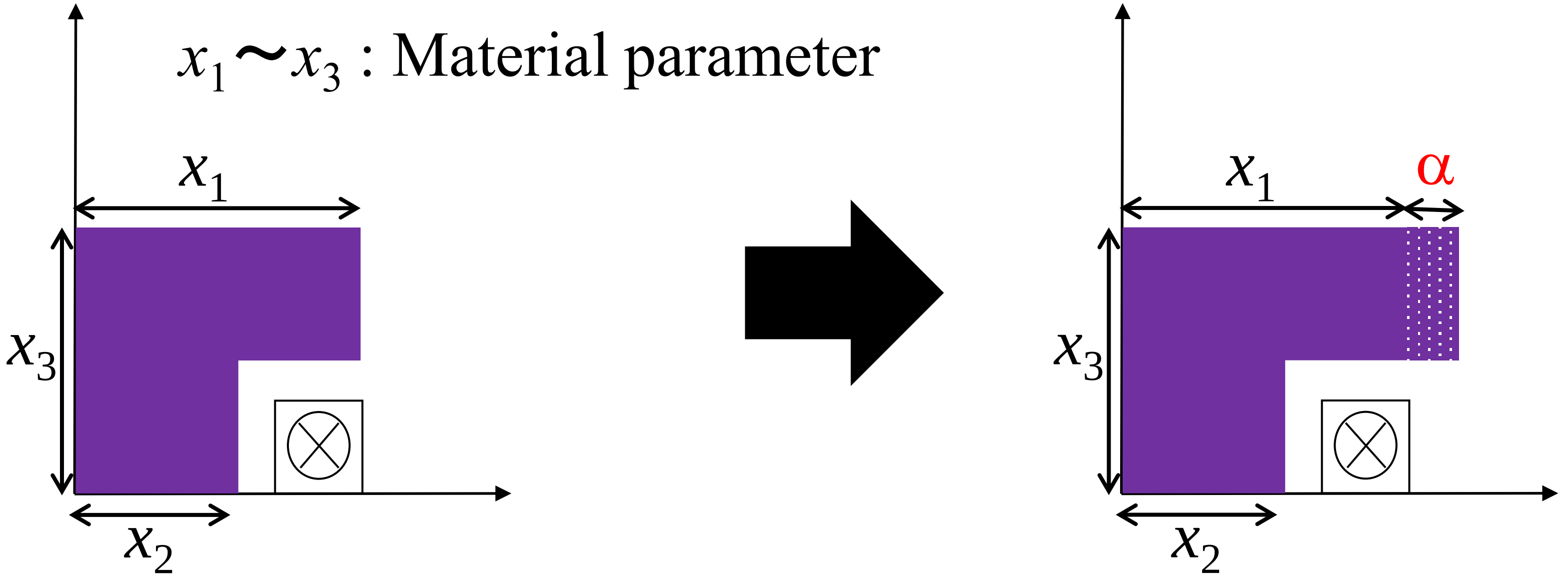
III. Optimization Results

IV. Conclusions

Background

- Shape optimization plays an important role in the development of electromagnetic devices.
- There are two approaches for shape optimizations, namely, parameter and topology optimizations.

Parameter optimization



Background

- Shape optimization plays an important role in the development of electromagnetic devices.
- There are two approaches for shape optimizations, namely, parameter and topology optimizations.

Parameter optimization

- Device shape are represented with design parameters
- Optimization is conducted by changing the parameters.

Dependence on experience and knowledge of engineers

Topology optimization

- This method seeks for the optimum solutions directly varying the material shape without design parameters.

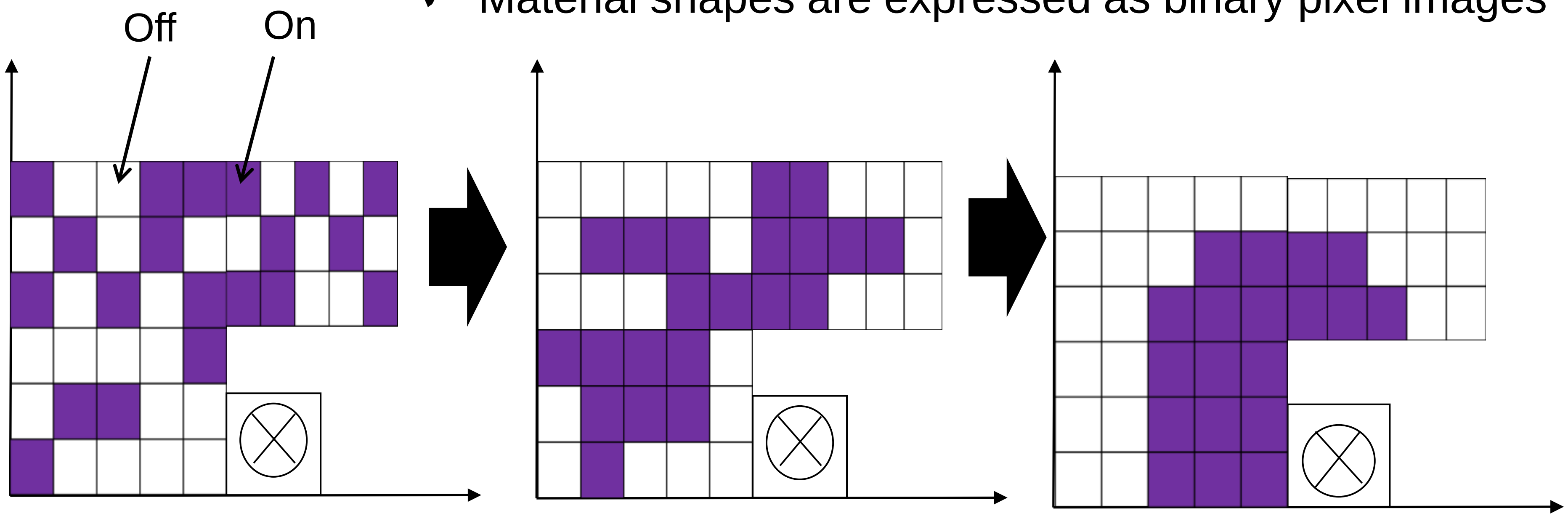
Find novel shape

Background

- In the topology optimization on-off and level-set methods are widely used.

On-Off Method

- ✓ Genetic Algorithm (GA) is widely employed for optimization process.
- ✓ Material shapes are expressed as binary pixel images



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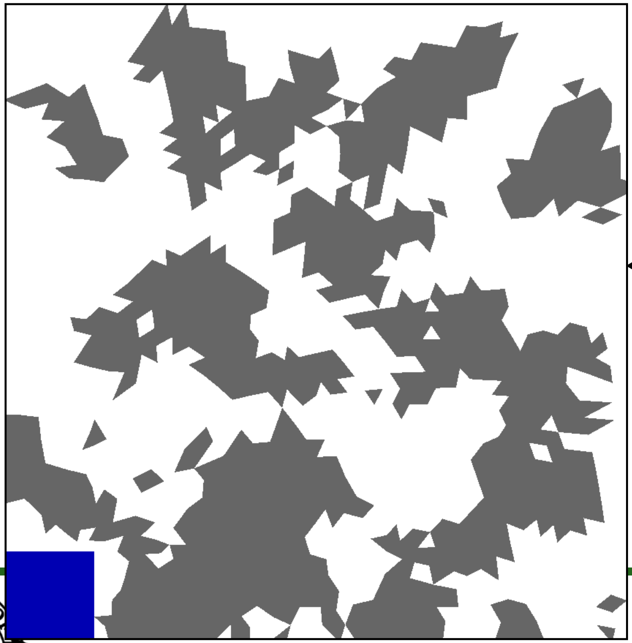


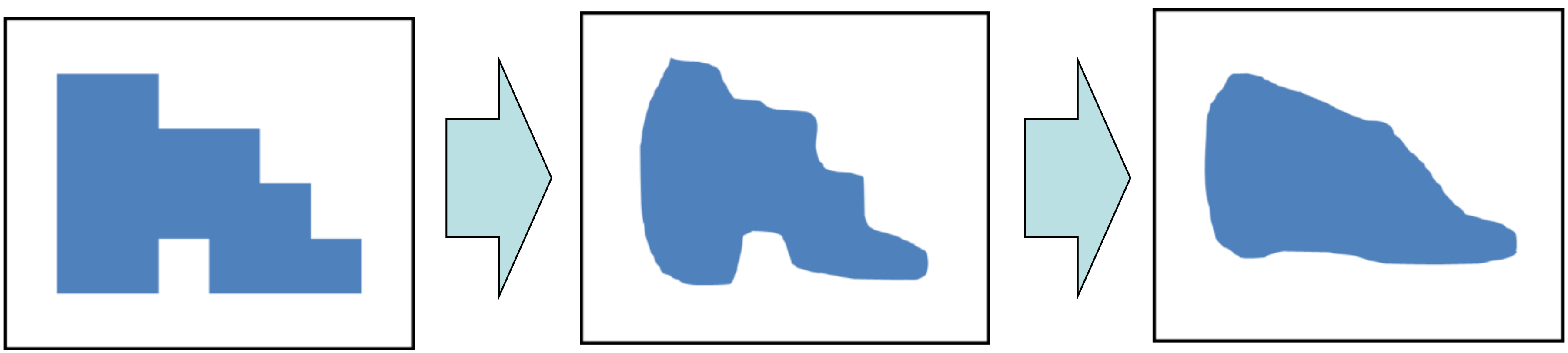
- We may obtain complicated shape because of huge search spaces.

Background

- In the topology optimization on-off and level-set methods are widely used.

Level set Method

- ✓ Material boundaries are expressed with level set function.
- ✓ We can have smooth boundaries and non-porous material region.
- ✓ This tends to fail into local optima because optimization is conducted based on gradient method.

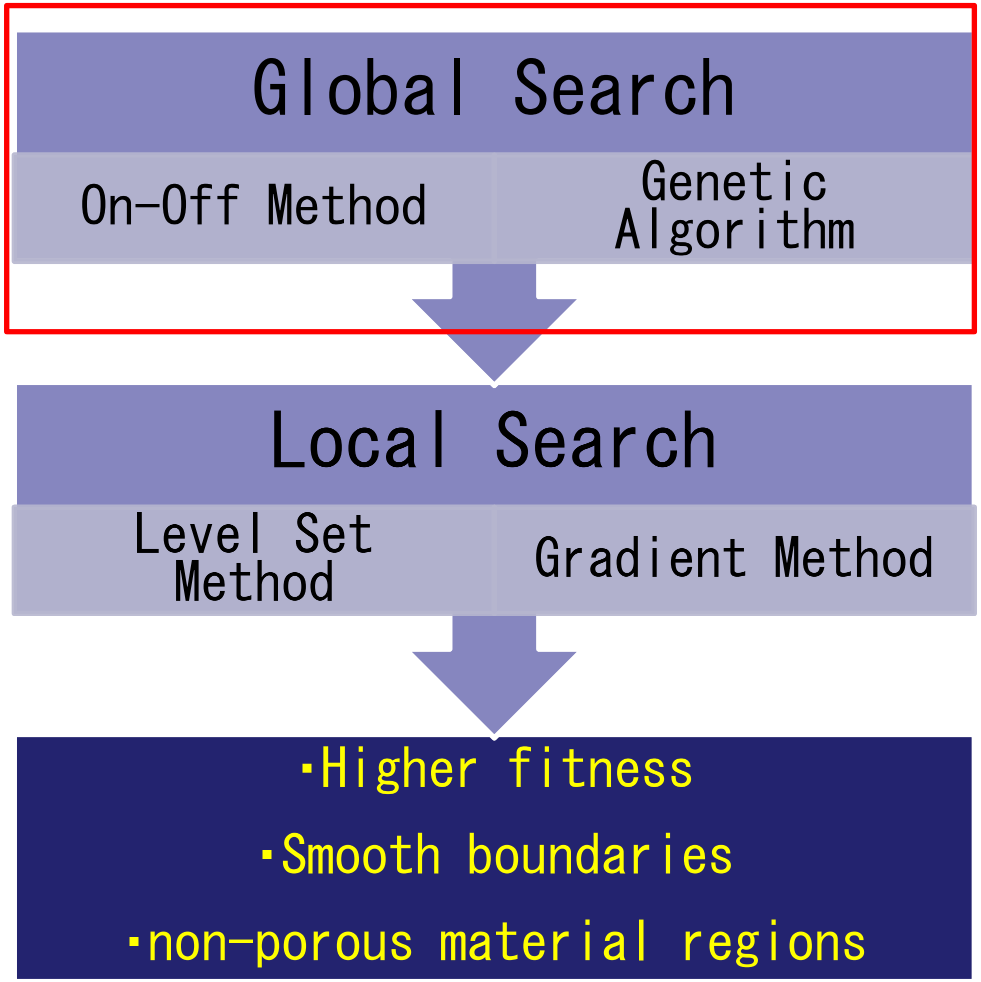


Purpose

- Present Method

First step is the global search.

- GA has good performance for the global search
- One solution is selected



Purpose

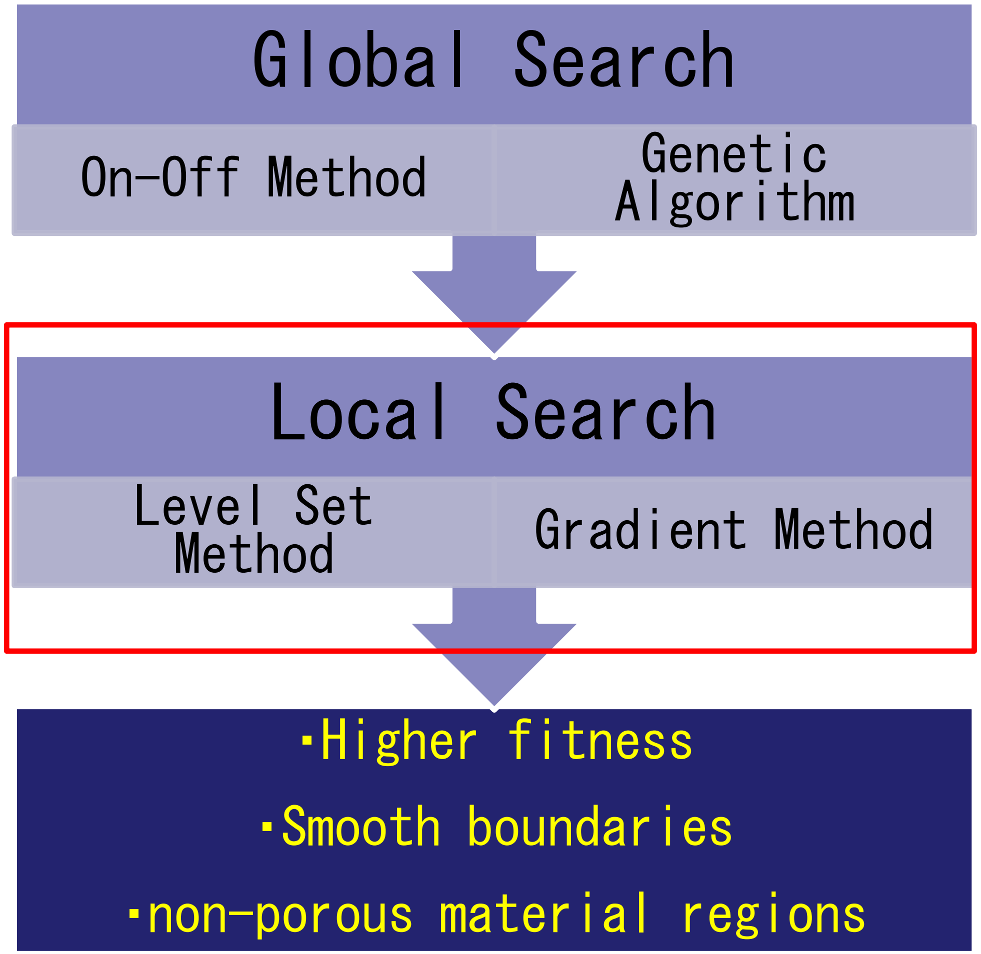
- Present Method

First step is the global search.

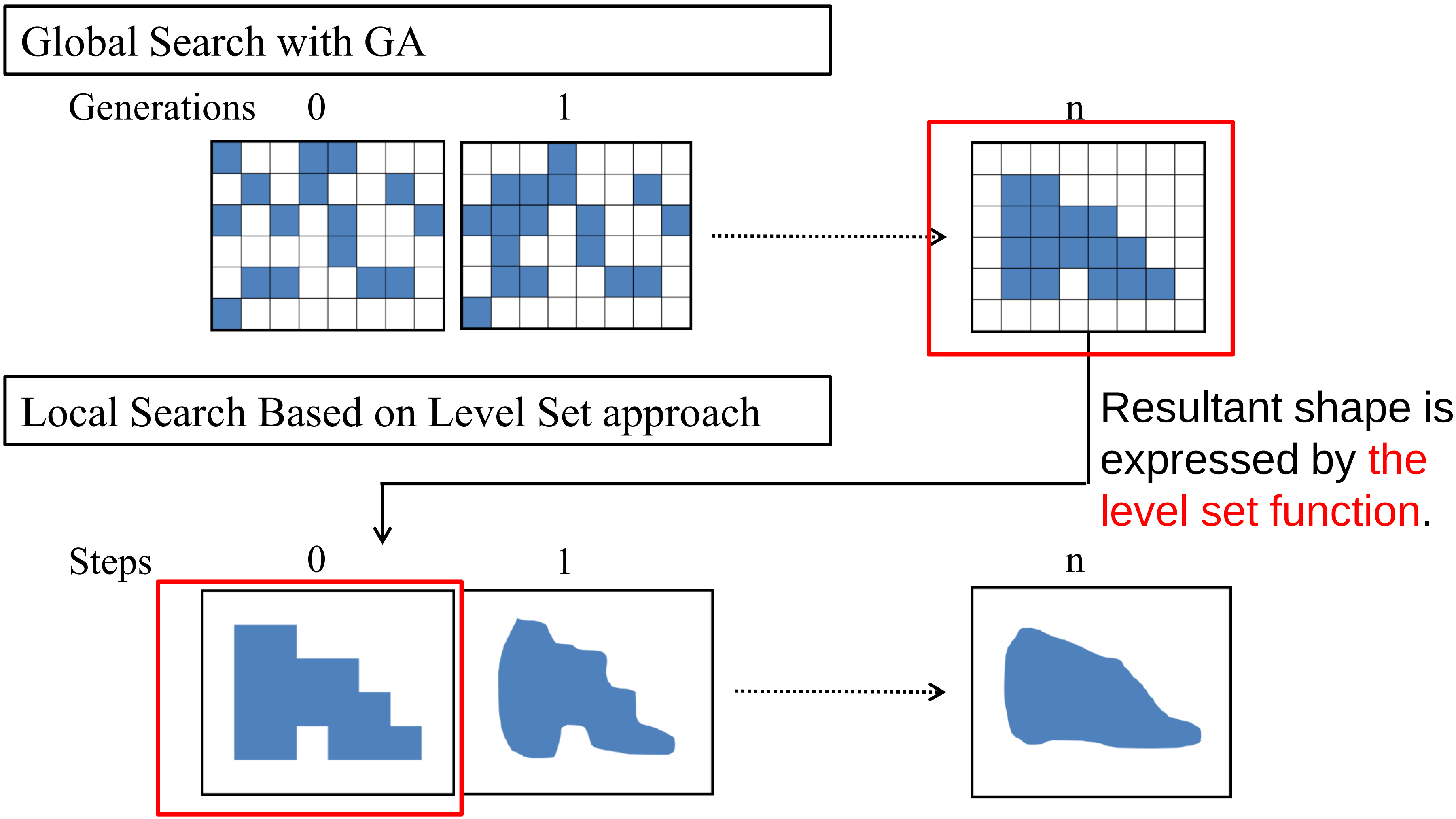
- GA has good performance for the global search
- One solution is selected

Second step is the local search,,,,

- The solution improved by level set method
- Smooth boundaries and non-porous material region

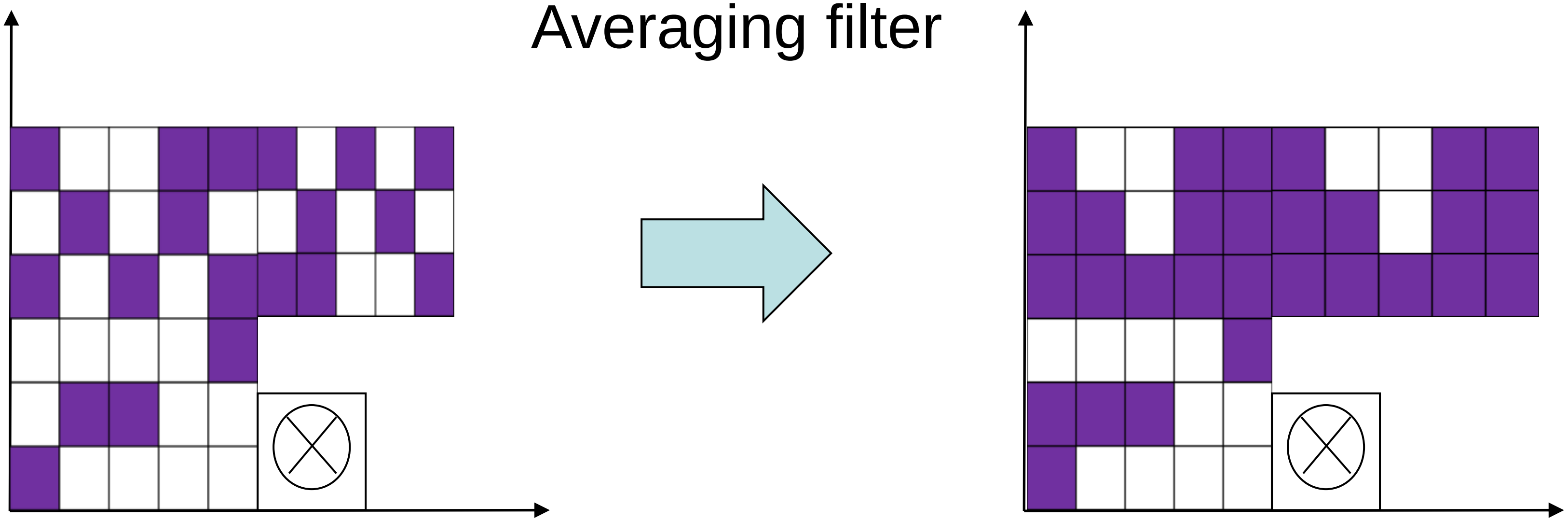


Outline of the present method



Global search method - On-Off Method -

- In order to suppress computational time, the micro genetic algorithms (μGA) is employed for optimization [1].
- To eliminate high frequency component, we applied the averaging filter for smoothing.



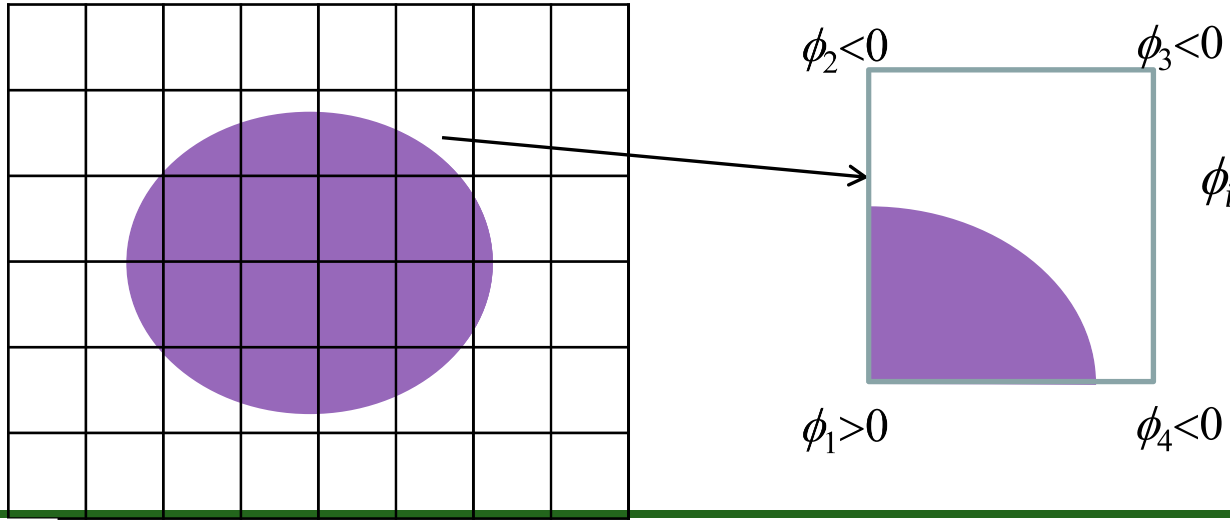
[1]. C. A. Coello and G. T. Pulido, "A micro-genetic algorithm for multiobjective optimization," EMO 2001, LNCS 1003, pp, 126-140, 2001.

Local search method - Level Set Method -

- Material shape is expressed in terms of the level set functions.
- The level set functions are defined on each node.
- The level set function of any point in each element calculates by interpolating.

- D : Design region
- Ω : Material region
- $\partial\Omega$: Material boundary
- $\boldsymbol{x}$: Point vector in D

$$\phi(\boldsymbol{x}) \begin{cases} > 0 & (\boldsymbol{x} \in \Omega) \\ = 0 & (\boldsymbol{x} \in \partial\Omega \cap D) \\ < 0 & (\boldsymbol{x} \in D \setminus \Omega) \end{cases}$$



ϕ_i : Level set function on each point

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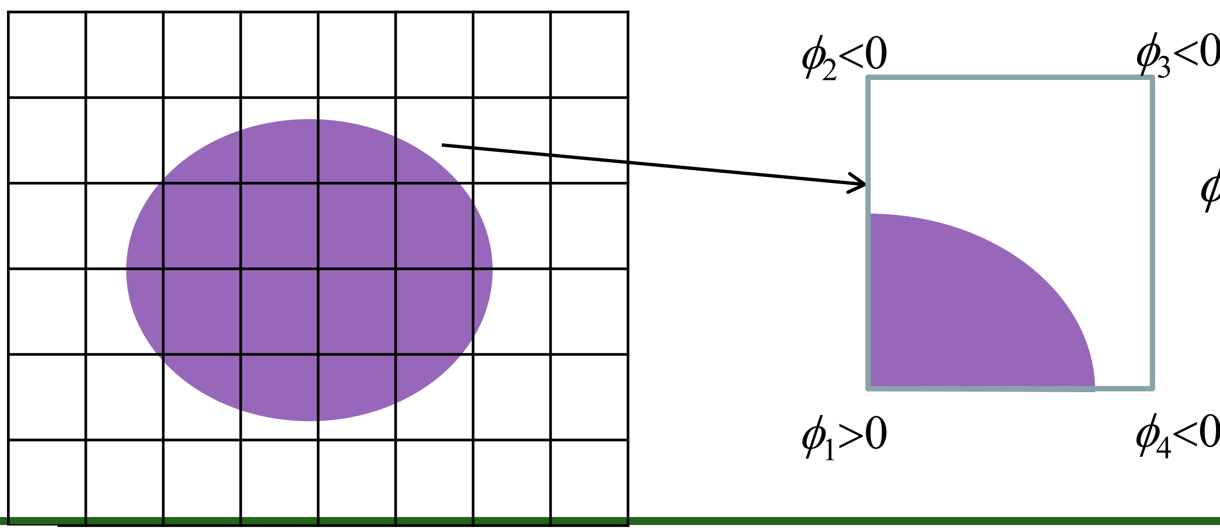
Distance function from material boundary



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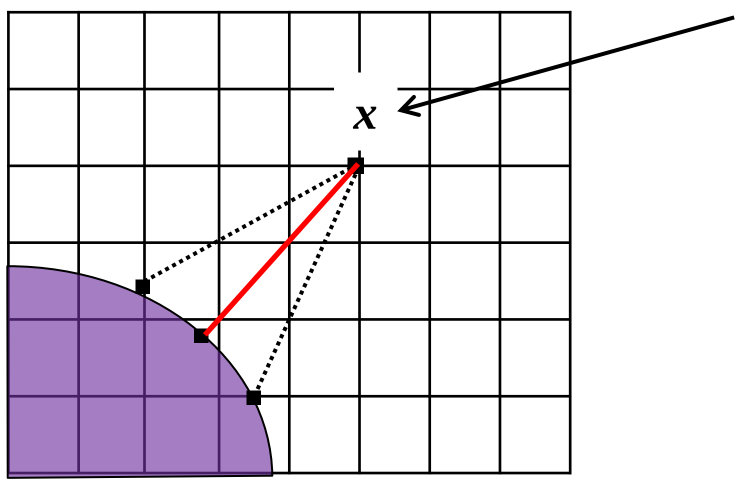
Level-Set method - Distance function -

- Level set function is defined by

$$\phi(\boldsymbol{x}) = \begin{cases} d(\boldsymbol{x}, \partial\Omega) & \boldsymbol{x} \in \Omega \\ 0 & \boldsymbol{x} \in \partial\Omega \\ -d(\boldsymbol{x}, \partial\Omega) & \boldsymbol{x} \notin \Omega \end{cases}$$

where d denotes the shortest distance between $\boldsymbol{x}$ and boundary.



- The value of level-set function ϕ is evaluated by

$$\phi(\boldsymbol{x}) = \min_{\boldsymbol{y} \in \partial\Omega} d(\boldsymbol{x}, \boldsymbol{y})$$

Level-Set method – In the optimization–

- Material shapes are expressed with using level-set function and optimization is conducted by changing them.
- Level-set function is updated to reduce the value of objective function as follows:

$$\phi_i^{n+1}(\boldsymbol{x}) = \phi_i^{n}(\boldsymbol{x}) + V_N$$

$$V_N = -\frac{df}{d\phi_i}$$

$$\frac{df}{d\phi_i} = \frac{\partial f}{\partial \phi_i} + \frac{\partial f}{\partial \boldsymbol{A}} \cdot \underbrace{\frac{\partial \boldsymbol{A}}{\partial \phi}}$$

- It is difficult to evaluate.

f : objective function

n : Iteration of optimization

V_N : update descent of the level-set functions

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- In order to evaluate the gradient, adjoint variable method is employed.

f : objective function
n : Iteration of optimization
V_N : update descent of the level-set functions

Adjoint variable method

- Differentiate f with respect to level-set function
  - a. Modified objective function defined by (1)
  - b. Differentiation of Eqn. (1) with respect to ϕ_i leads to (2)
  - c. Update the level-set function using V_N

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a. $\hat{f} = f + \boldsymbol{z}^T(K\boldsymbol{A} - \boldsymbol{b})$ (1)

$\hat{f} \simeq f$ if $\boldsymbol{A}$ exactly satisfies $K\boldsymbol{A}=\boldsymbol{b}$

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b. $$\frac{d\hat{f}}{d\phi_i} = \frac{\partial f}{\partial \phi_i} + z^T \frac{\partial K}{\partial \phi_i}\boldsymbol{A} + \left(\boldsymbol{z}^T K + \frac{\partial f}{\partial \boldsymbol{A}}\right)\frac{dA}{d\phi_i} \quad (2)$$

In order to avoid evaluating this

Adjoint variable method

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⇨ 0

$$K\boldsymbol{z} = -\frac{\partial f}{\partial \boldsymbol{A}}^T$$

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$\Rightarrow$ 0

c. $$\frac{df}{d\phi} = \frac{d\hat{f}}{d\phi_i} = \frac{\partial f}{\partial \phi_i} + z^T \frac{\partial K}{\partial \phi_i}\boldsymbol{A}$$

Numerical exmple 1 - IPM-Motor -

- The purpose of this optimization is to maximize the torque average and minimize the torque ripple.
- Shape of the flux barrier in the rotor is optimized.

■ Optimization problem

$$F = -T_{AVG} + W * T_{rip} \rightarrow Min.$$

where

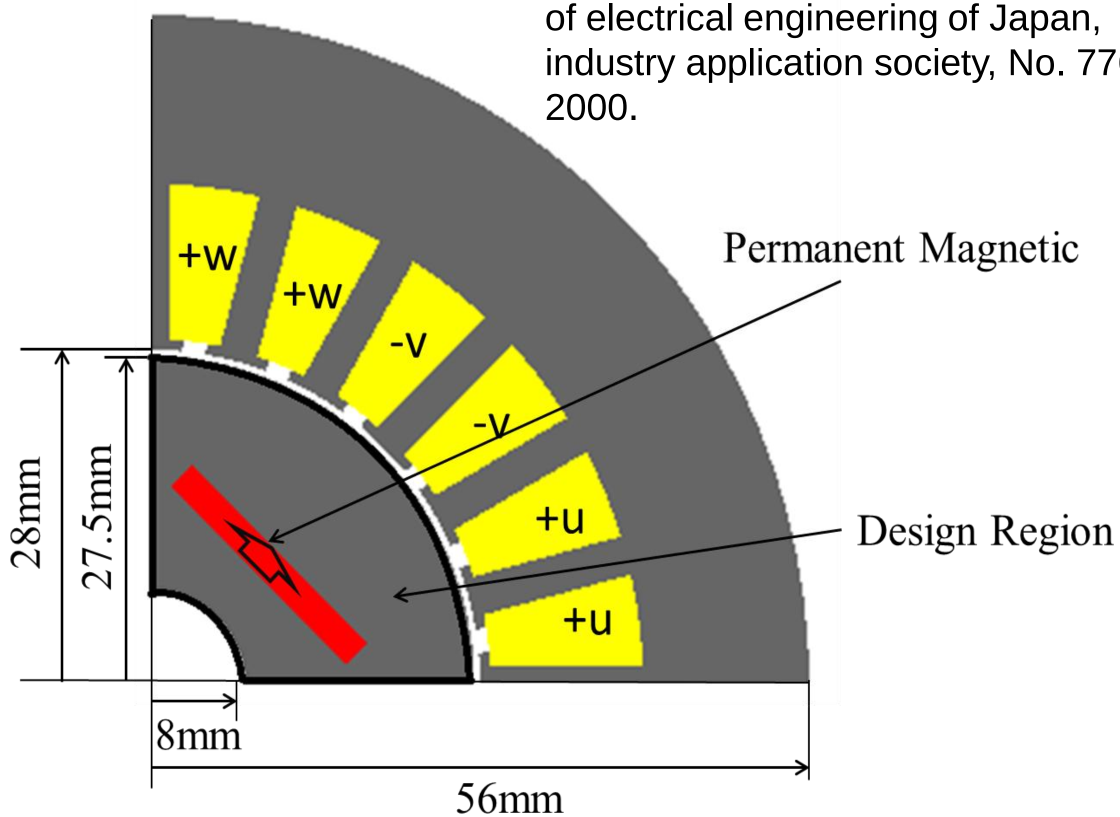
$$T_{rip} = \frac{T_{max} - T_{min}}{T_{AVG}}$$

T_{AVG} : Torque average [Nm]

T_{rip} : Torque ripple

W : Weighting coefficient

[3]. Technical report of the institute of electrical engineering of Japan, industry application society, No. 776, 2000.



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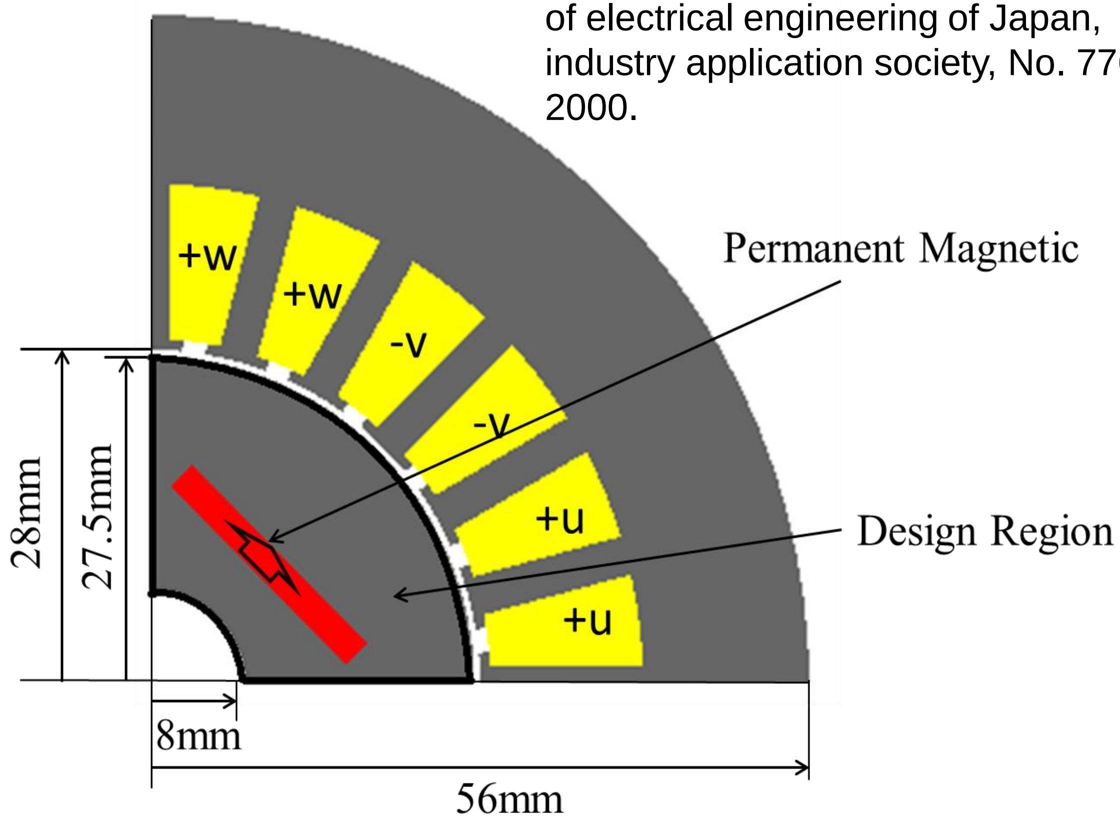
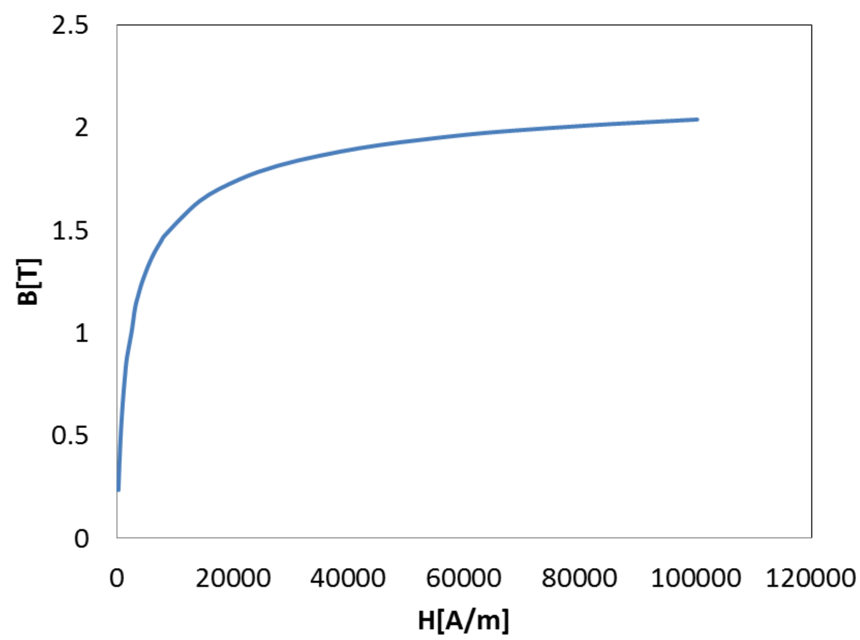


IPM-Motor - Analysis conditions -

Rotation speed (rpm)	3000
Armature current (A)	600
Phase of current (degree)	20
Residual flux density of PM (T)	1.0
Width of teeth (mm)	3.3
Length of Coil (mm)	25.9
Thickness of PM (mm)	2.5
Width of PM (mm)	21



- ✓ Computational time : 10 [h]
- ✓ Number of unknown in FE analysis : about 2,000

Computational environment

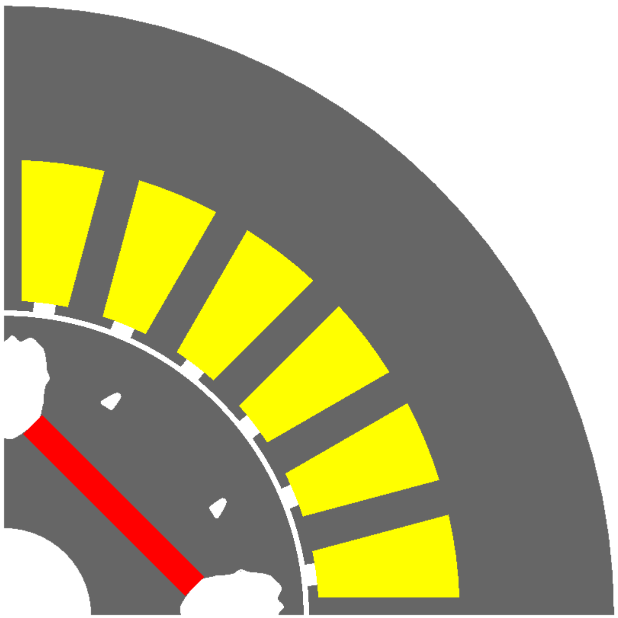
・CPU : Xeon X5660(6-Core 2.8GHz, 6×256KB+12MB,1333MHz)×2

・Main memory : 12GByte

Optimization results

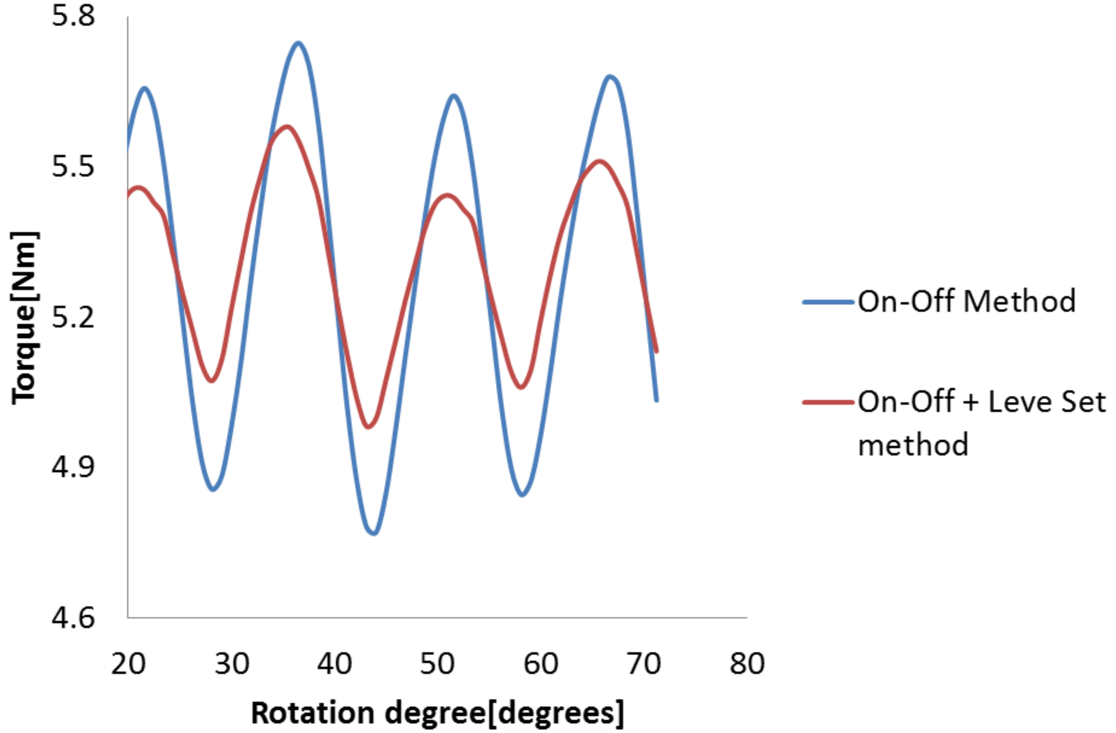
On-Off method	On-Off + Level Set method

Torque average (Nm)	5.280
Torque ripple	0.184
Objective function	-0.806



Torque average (Nm)	5.309
Torque ripple	0.112
Objective function	-1.018

Optimization results



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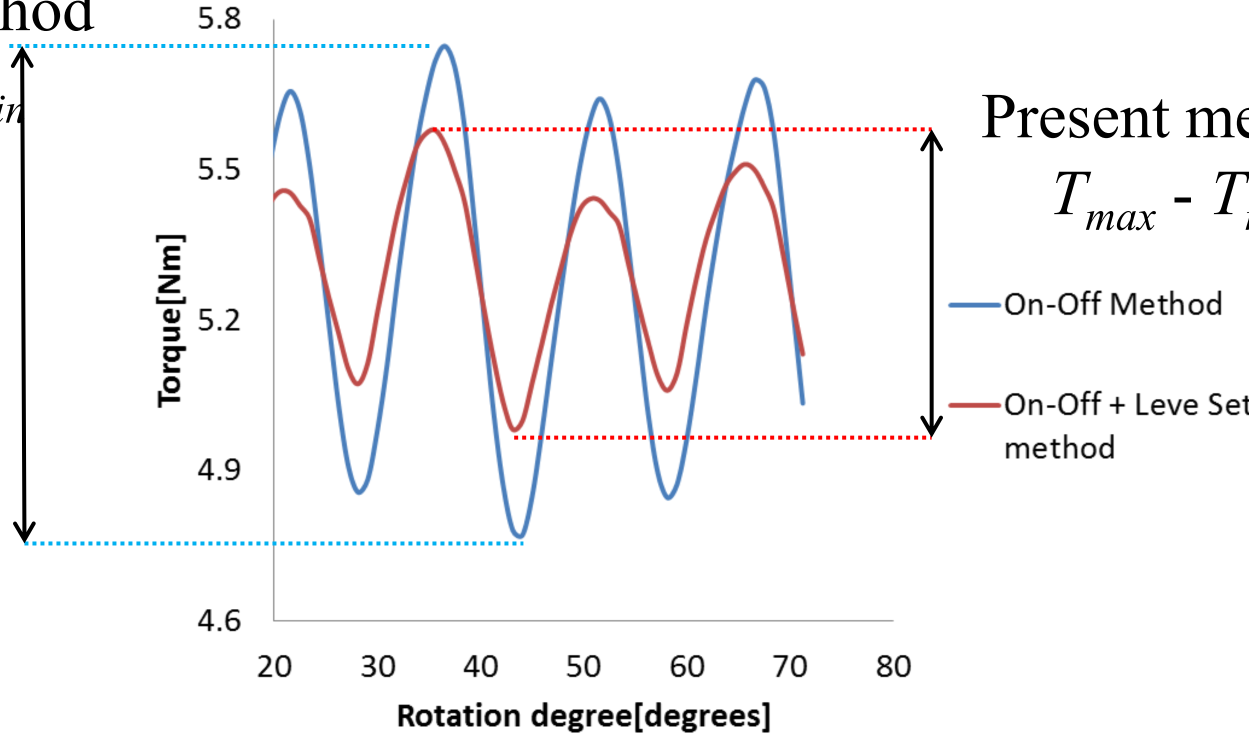
Optimization results

On-Off method

T_{max} - T_{min}



Present method

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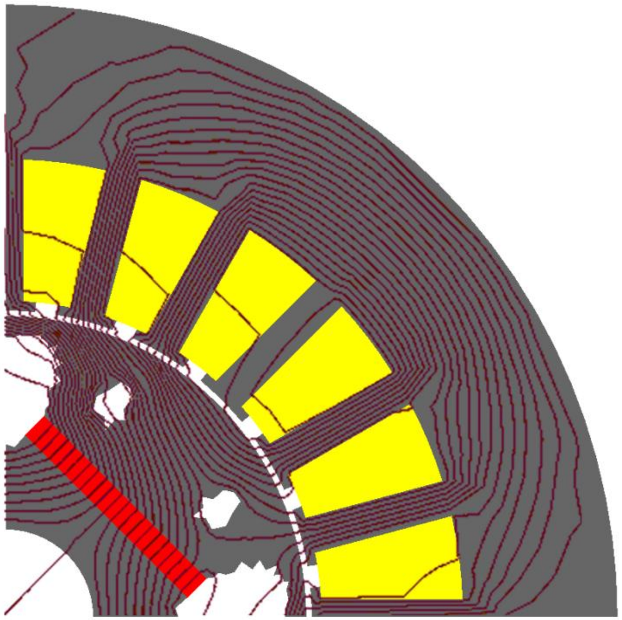
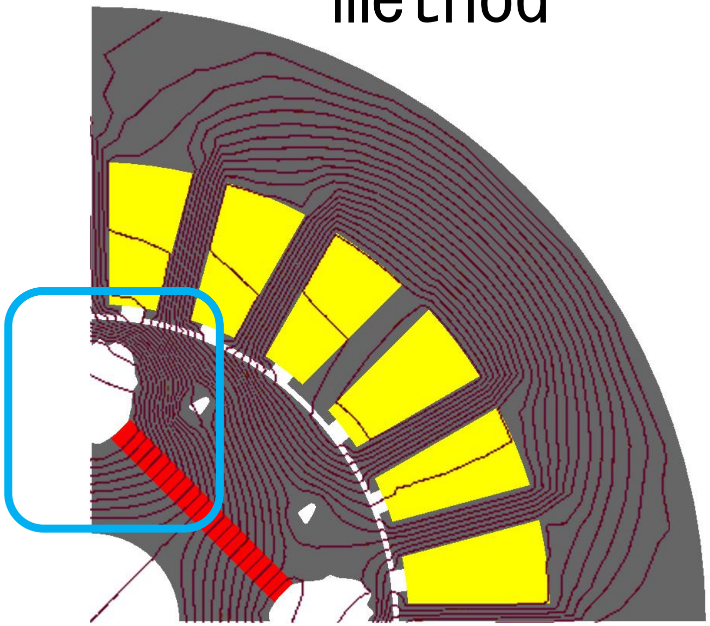
Optimization results - Flux distribution -

On-Off method	On-Off + Level Set method





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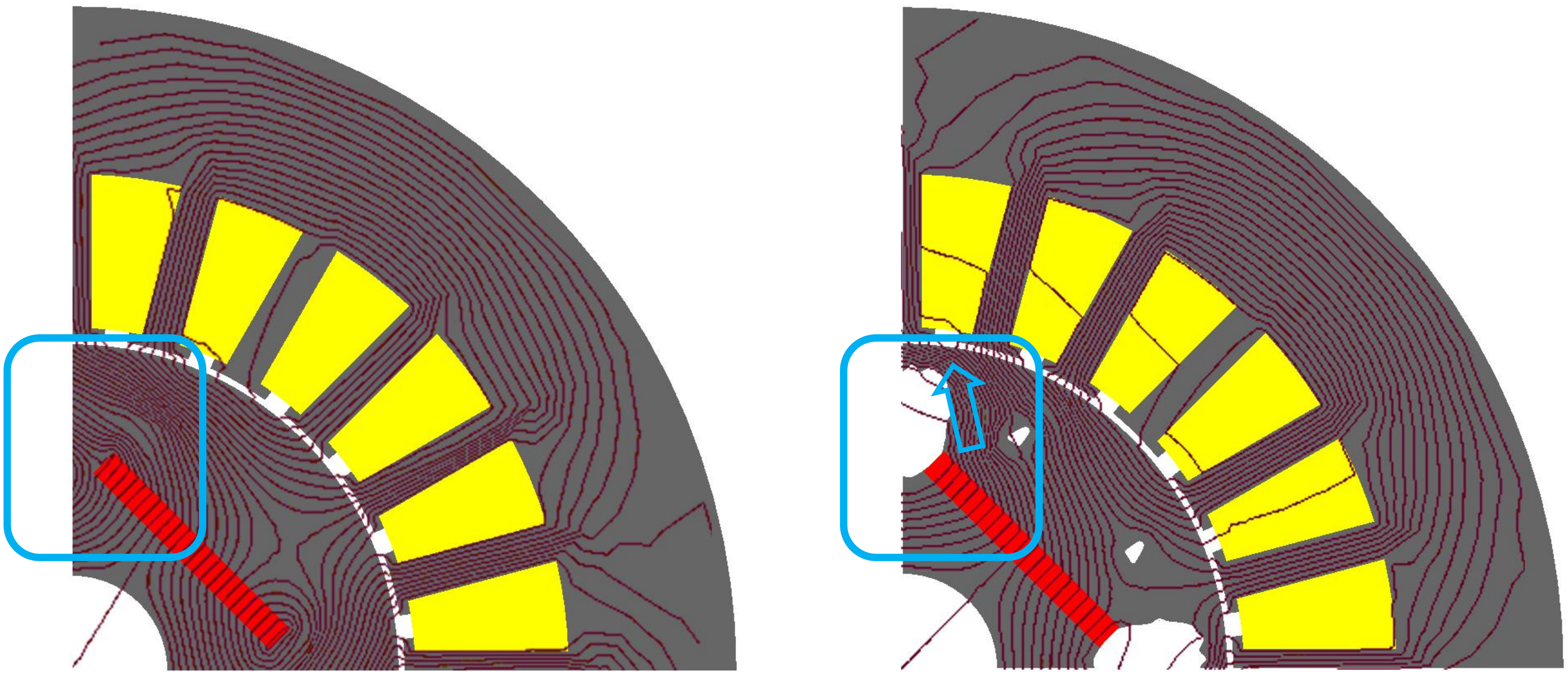
Optimization results - Flux distribution -

Non Flux Barrier	Flux Barrier



- Due to the flux barriers, magnetic flux goes to the rotor surface.

Numerical example 2 – Magnetic shield –

- The present method is applied to magnetic shield model shown in figure.
- The purpose of this optimization is to minimize the flux density in Evaluated region and core volume created in design region.

■Optimization Problem

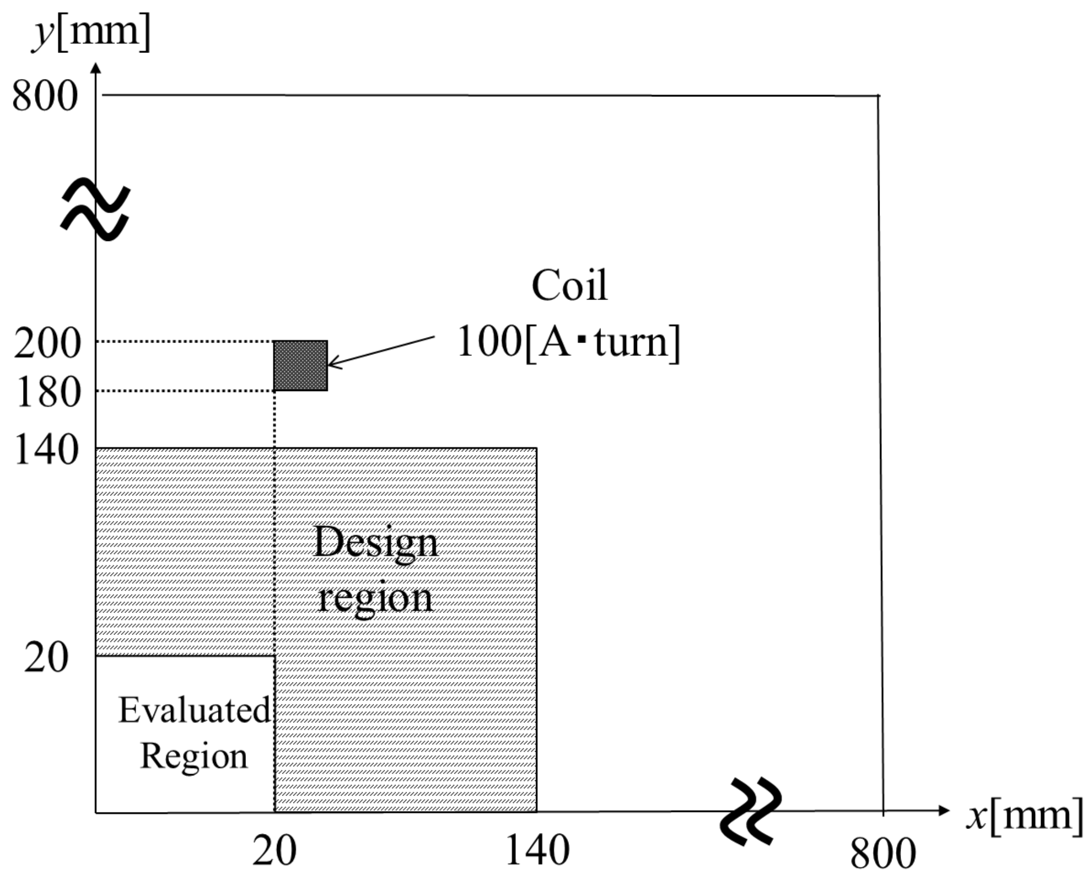
$$F(\phi) = W_M \frac{|\boldsymbol{B}|_{average}}{10^{-5}} + \frac{S}{S_{design}} \rightarrow Min.$$

W_M : weighting coefficient

S : volume of the core

S_{design} : volume of the design region

$\boldsymbol{B}$: flux density of the evaluated region



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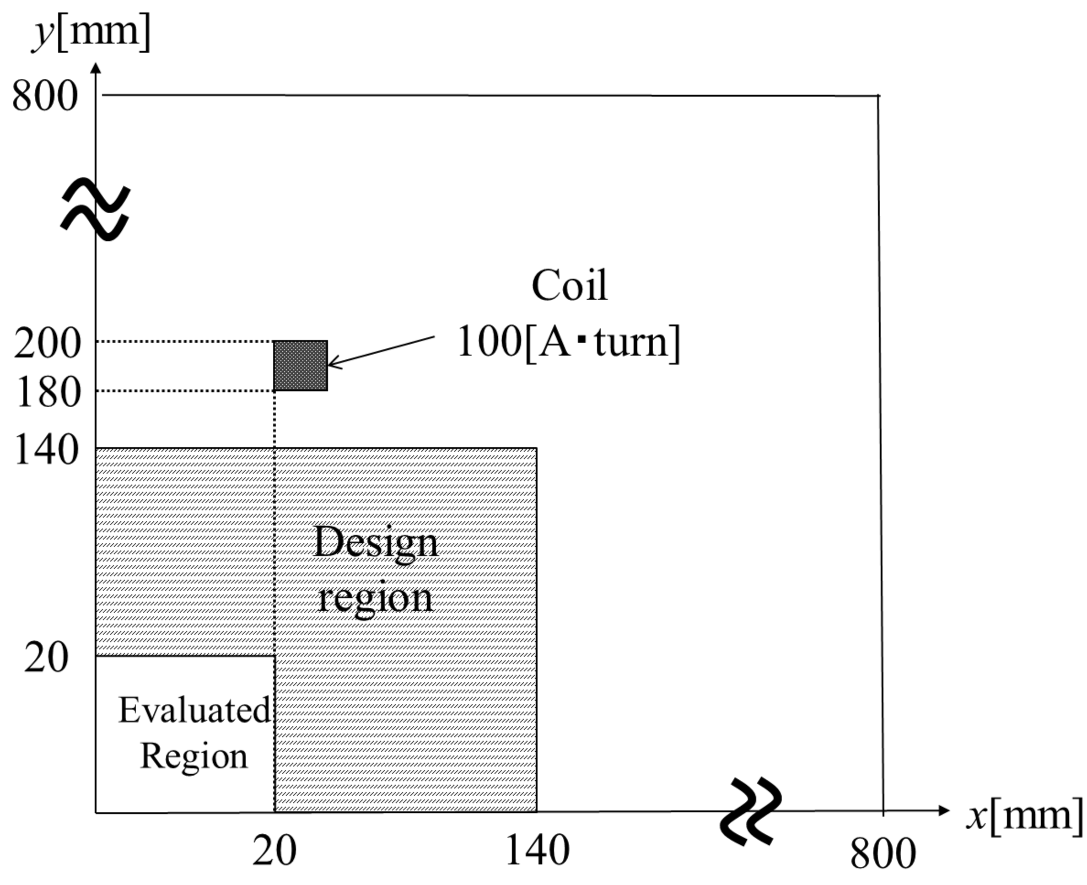


Magnetic shield − Optimization parameter −

Number of elements in design region	2,488
Number of elements in analysis region	5,052
Generation of global search (μGA)	200
Generation of local search (Level Set)	200
Weighting coefficient : W_M	0.2

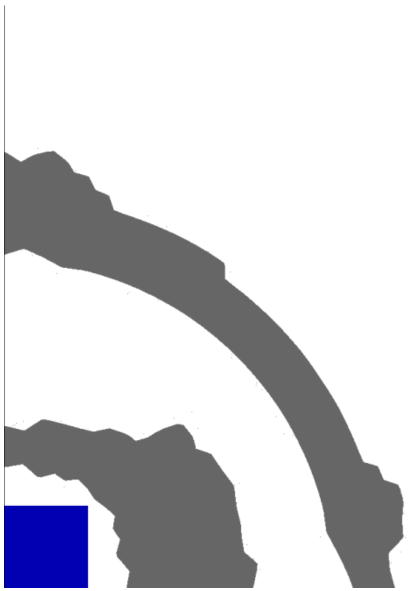
✓ Computational time ：2[h]
✓ Number of unknown in FE analysis : about

Computational environment

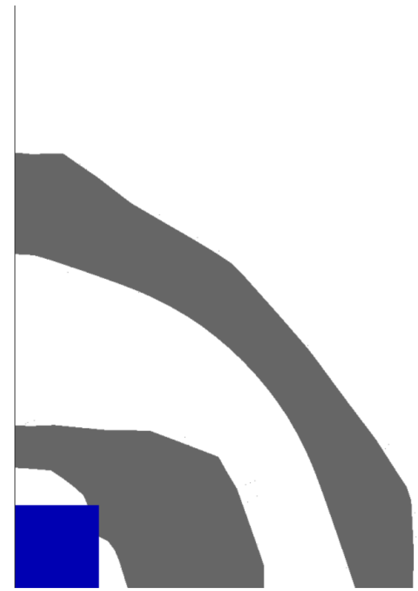
・CPU ： Xeon X5660(6-Core 2.8GHz, 6×256KB+12MB,1333MHz)×2
・Main memory ： 12GByte

Magnetic shield

On-Off method	On-Off + Level Set method



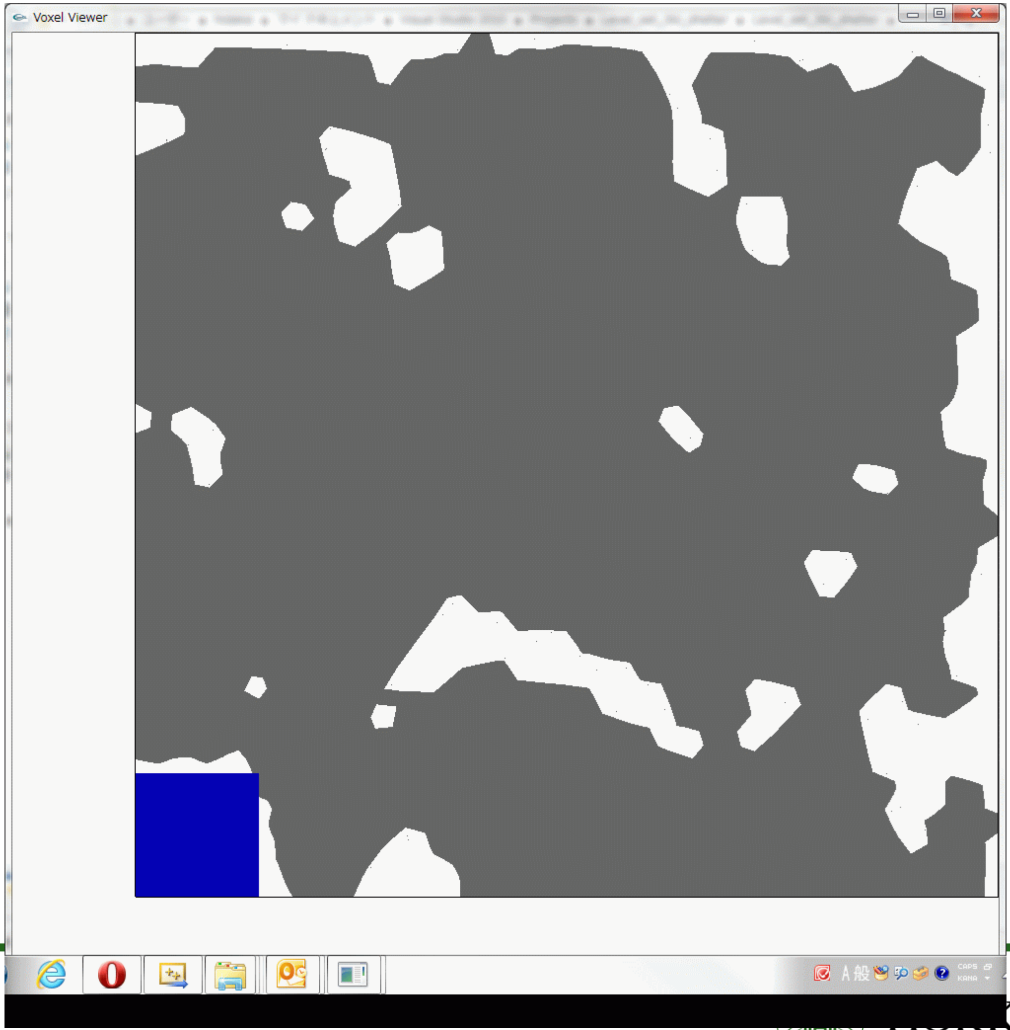
$\|\boldsymbol{B}\|_{average}/10^{-5}$	0.124
Volume of the core (cm^2)	2.736
Objective function	0.0678



$\|\boldsymbol{B}\|_{average}/10^{-5}$	0.0990
Volume of the core (cm^2)	2.776
Objective function	0.0631

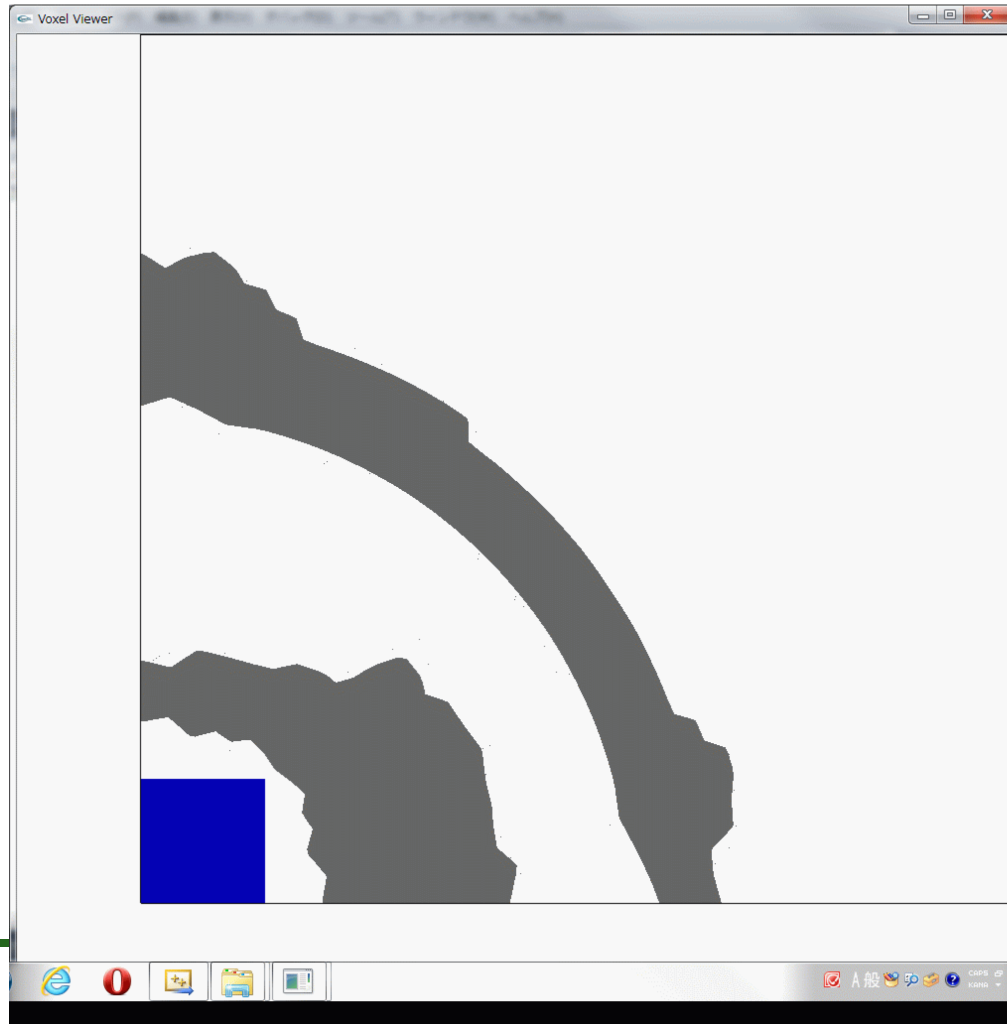
Magnetic shield

Global search



Magnetic shield

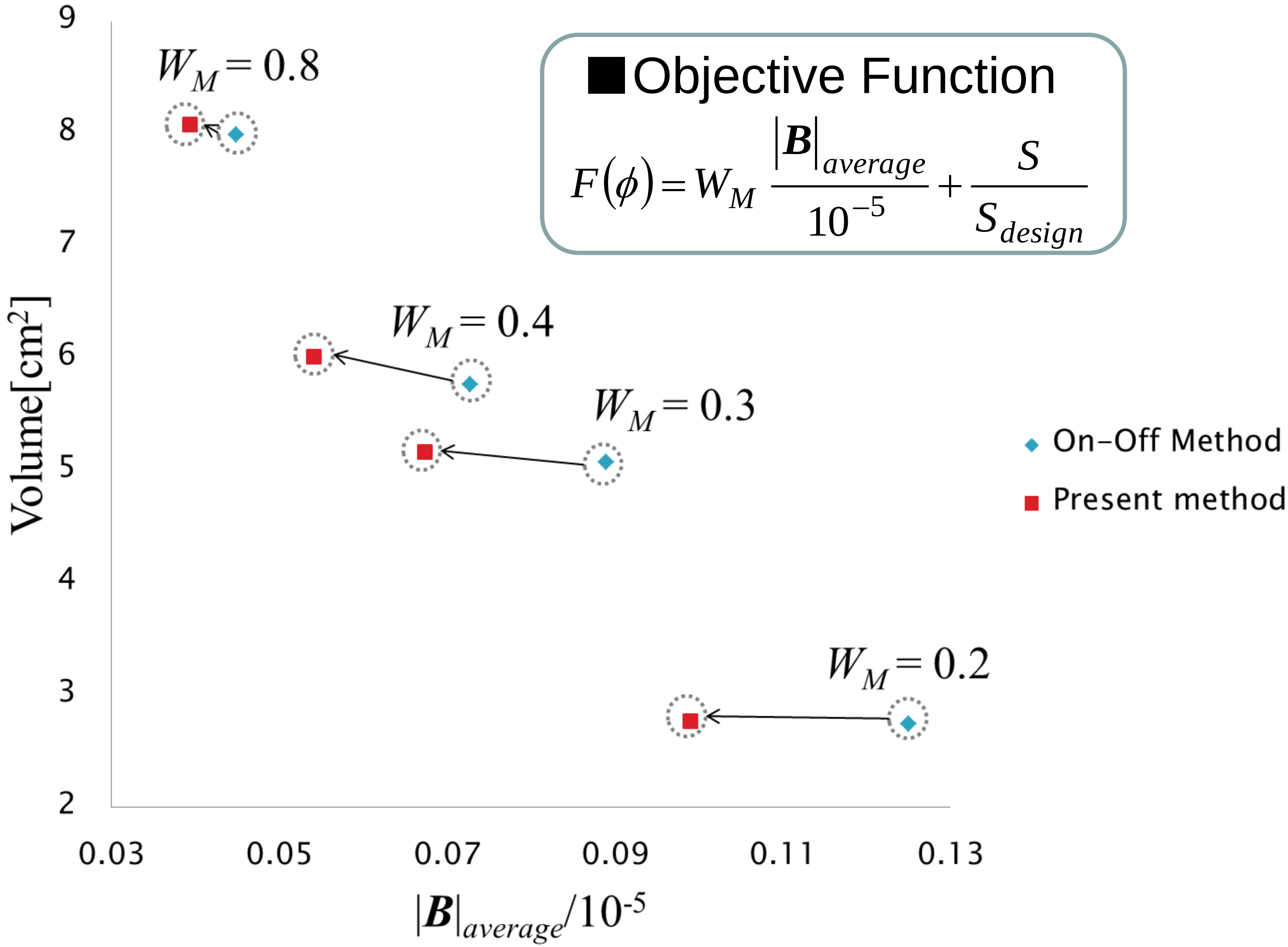
Local search



Magnetic shield

■Objective Function

$$F(\phi) = W_M \frac{|\boldsymbol{B}|_{average}}{10^{-5}} + \frac{S}{S_{design}}$$



Magnetic shield

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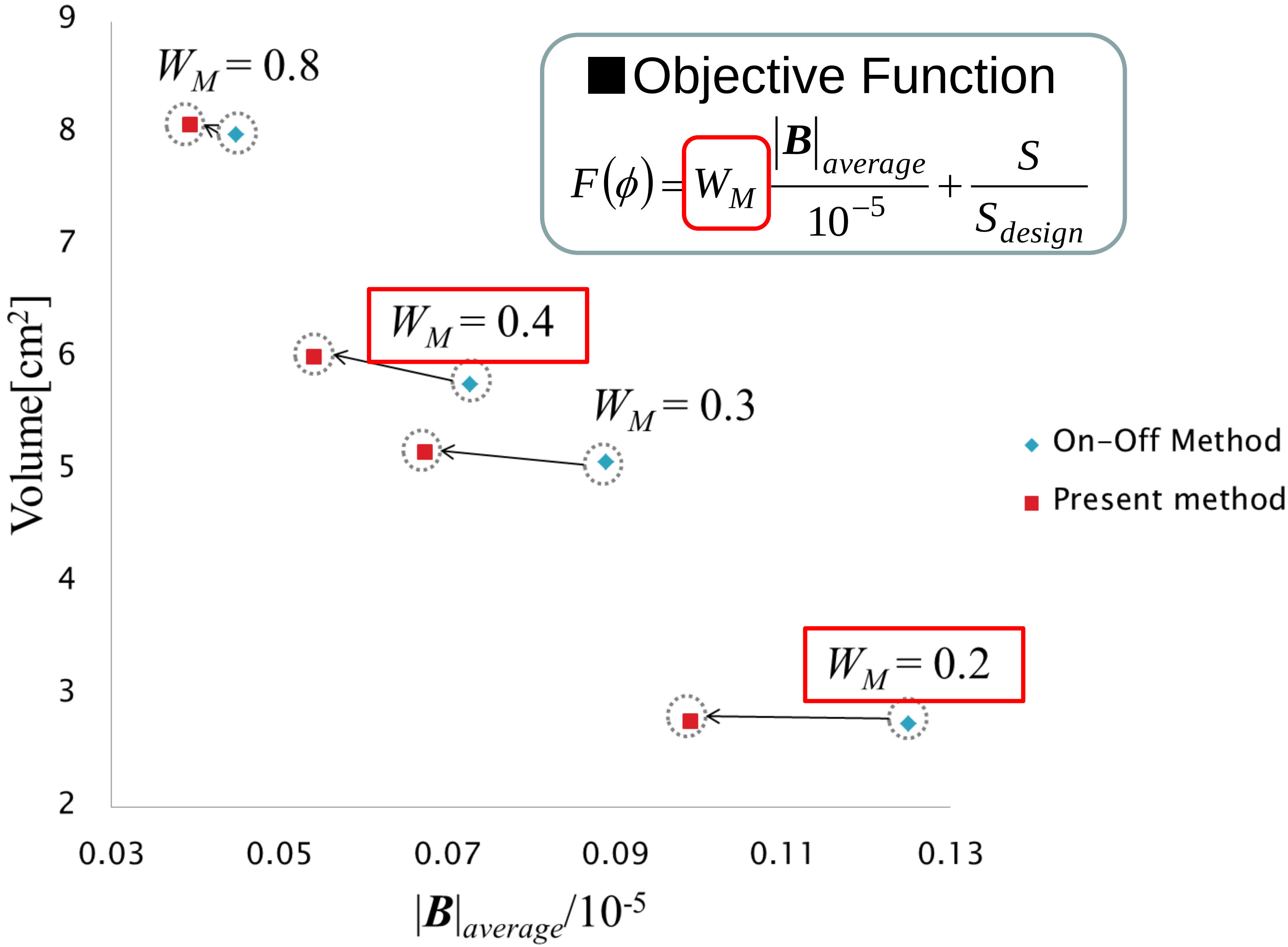
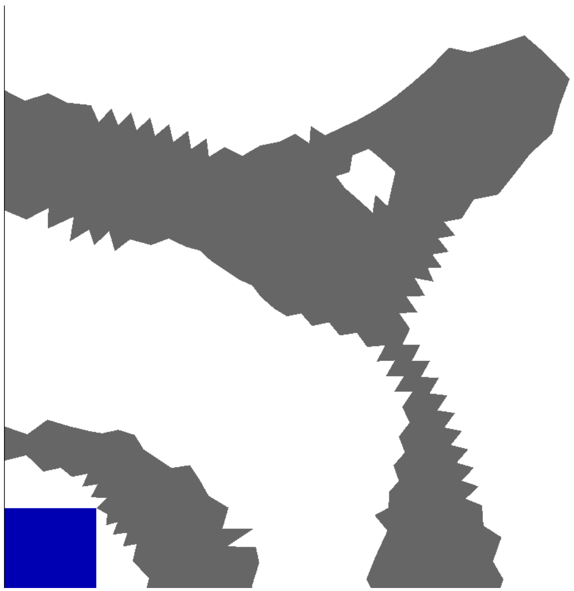
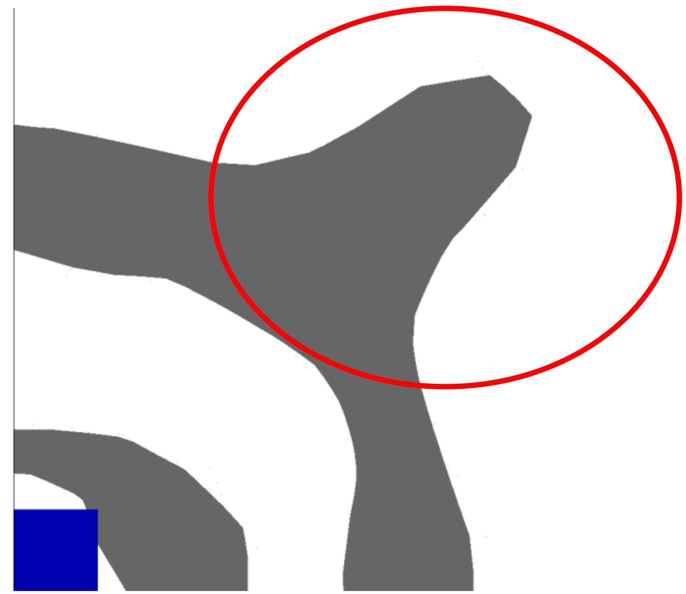


Magnetic shield (W_M=0.4)

On-Off method	On-Off + Level Set method



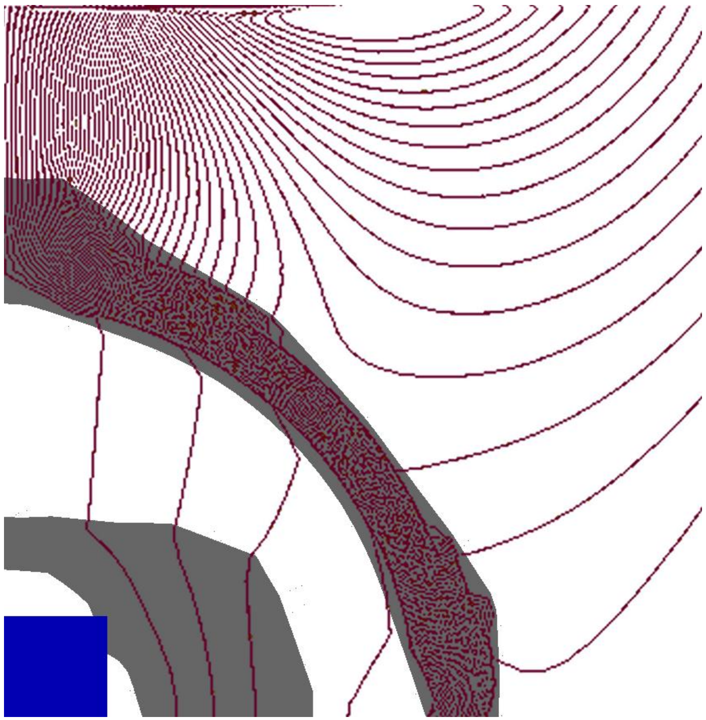


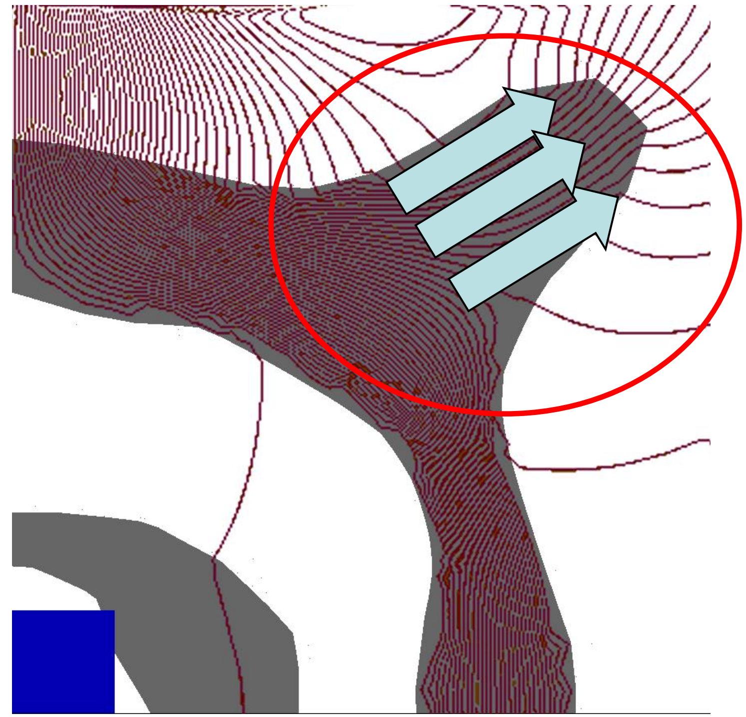
$\lvert \boldsymbol{B} \rvert_{average}/10^{-5}$	0.0727
Volume of the core (cm^2)	5.864
Objective function	0.121

$\lvert \boldsymbol{B} \rvert_{average}/10^{-5}$	0.0541
Volume of the core (cm^2)	6.016
Objective function	0.116

Magnetic shield – Consideration of Branch –

Non protuberance	A protuberance





- Due to protuberance occurs from out shield, flux goes to outside of the shield.

Conclusions

- we present a new topology optimization method which based on the on-off and level set methods.
- In order to test this method, it is applied to numerical examples.
- The results show the present method can effectively find optimal solution which have better performances.

Future works

- Applied to the 3-dimentional problems and other devices
- Introduce the multi-objective GA